

Solve each polynomial equation by factoring.

$$1. -3x^4 + 6x^3 + 105x^2 = 0$$

$$-3x^2(x^2 - 2x - 35) = 0$$

$$-3x^2(x-7)(x+5) = 0$$

\downarrow \downarrow \downarrow
 $x=0$ $x=7$ $x=-5$

$$2. 8x^7 - 56x^6 + 96x^5 = 0$$

$$8x^5(x^2 - 7x + 12) = 0$$

$$8x^5(x-4)(x-3) = 0$$

\downarrow \downarrow \downarrow
 $x=0$ $x=4$ $x=3$

Identify the roots of each equation. State the multiplicity of each root. *Look at graph

3. $x^3 + 6x^2 + 12x - 8 = 0$

4. $x^3 + 10x^2 + 32x + 32 = 0$

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$x = -4$ mult. of 2 $x = -2$ mult. of 1

Identify all the real roots of each equation.

5. $x^3 + 2x^2 - 48x = 0$

$$x(x^2 + 2x - 48) = 0$$

$$x(x+8)(x-6) = 0$$

$x = 0, -8, 6$

6. $x^4 - 13x^3 + 55x^2 - 81x + 18 = 0$ From graph: 3 + 6

3		-13	55	-81	18	
		↓	3	-30	75	-18
6		1	-10	25	-6	Lor
		↓	6	-24	6	
		1	-4	1	Lor	

$x^2 - 4x + 1$

$x = 4 \pm \frac{\sqrt{16 - 4(1)(1)}}{2(1)}$

$x = 2 \pm \sqrt{3}$

$x = 3, 6, 2 \pm \sqrt{3}$

7. $6x^3 + 12x^2 - 18x = 0$

$$6x(x^2 + 2x - 3) = 0$$

$$6x(x+3)(x-1) = 0$$

$x = 0, -3, 1$

8. $x^4 + 8x^3 + 7x^2 - 22x + 6 = 0$ From graph: -3, 1

-3		8	7	-22	6	
		↓	-3	-15	24	-6
1		1	5	-8	2	Lor
		↓	1	6	-2	
		1	6	-2	Lor	

$x^2 + 6x - 2$

$x = -6 \pm \frac{\sqrt{36 - 4(1)(-2)}}{2(1)}$

$x = -3 \pm \sqrt{11}$

$x = -3, 1, -3 \pm \sqrt{11}$

Write the simplest polynomial function with the given roots.

9. $-\frac{3}{4}, 6,$ and -1

$$(x + \frac{3}{4})(x - 6)(x + 1)$$

$$x^2 - 6x + \frac{3}{4}x - \frac{9}{2}$$

$$(x^2 - \frac{21}{4}x - \frac{9}{2})(x + 1)$$

$$f(x) = x^3 - \frac{21}{4}x^2 - \frac{9}{2}x + x^2 - \frac{21}{4}x - \frac{9}{2}$$

$f(x) = x^3 - \frac{17}{4}x^2 - \frac{39}{4}x - \frac{9}{2}$

10. $-5i, 2,$ and $7, 5i$

$$(x + 5i)(x - 5i)(x - 2)(x - 7)$$

$$(x^2 + 25)(x^2 - 9x + 14)$$

$$f(x) = x^4 - 9x^3 + 14x^2 + 25x^2 - 225x + 350$$

$f(x) = x^4 - 9x^3 + 39x^2 - 225x + 350$

11. $-i, -3,$ and $-1, i$

$$(x + i)(x - i)(x + 3)(x + 1)$$

$$(x^2 + 1)(x^2 + 4x + 3)$$

$$f(x) = x^4 + 4x^3 + 3x^2 + x^2 + 4x + 3$$

$f(x) = x^4 + 4x^3 + 4x^2 + 4x + 3$

12. $2i, 4,$ and $\sqrt{6}, -2i, -\sqrt{6}$

$$(x - 2i)(x + 2i)(x - \sqrt{6})(x + \sqrt{6})(x - 4)$$

$$(x^2 + 4)(x^2 - 6)(x - 4)$$

$$(x^4 - 2x^2 - 24)(x - 4)$$

$$f(x) = x^5 - 2x^3 - 24x - 4x^4 + 8x^2 + 96$$

$f(x) = x^5 - 4x^4 - 2x^3 + 8x^2 - 24x + 96$

Solve each equation by finding all roots.

13. $4x^4 - 8x^3 - 3x^2 - 18x - 27 = 0$ $-1, 3$

$$\begin{array}{r|rrrrr} -1 & 4 & -8 & -3 & -18 & -27 \\ & \downarrow & -4 & 12 & -9 & 27 \\ 3 & 4 & -12 & 9 & -27 & \text{LoV} \\ & \downarrow & 12 & 0 & 27 & \\ 4 & 0 & 9 & & & \text{LoV} \end{array}$$

 $x^2 = -\frac{9}{4}$

 $4x^2 + 9 = 0$

 $4x^2 = -9$

 $x = -1, 3, \pm \frac{3i}{2}$

14. $x^4 + 3x^3 - x^2 + 9x - 12 = 0$ $-4, 1$

$$\begin{array}{r|rrrrr} -4 & 1 & 3 & -1 & 9 & -12 \\ & \downarrow & -4 & 4 & -12 & 12 \\ -1 & 1 & -1 & 3 & -3 & \\ & \downarrow & 1 & 0 & 3 & \\ 1 & 0 & 3 & & & \text{LoV} \end{array}$$

 $x = \pm i\sqrt{3}$

 $x^2 + 3 = 0$

 $x^2 = -3$

 $x = -4, 1, \pm i\sqrt{3}$

15. $x^4 - 3x^3 - 8x^2 + 22x - 24 = 0$ $-3, 4$

$$\begin{array}{r|rrrrr} -3 & 1 & -3 & -8 & 22 & -24 \\ & \downarrow & -3 & 18 & -30 & 24 \\ 4 & 1 & -6 & 10 & -8 & \text{LoV} \\ & \downarrow & 4 & -8 & 8 & \\ 1 & -2 & 2 & & & \text{Lo} \end{array}$$

 $x = 2 \pm \frac{\sqrt{4-4(1)(2)}}{2(1)}$

 $x = -3, 4, 1 \pm i$

 $x^2 - 2x + 2$

16. $x^3 + 6x^2 + 4x + 24 = 0$ -6

$$\begin{array}{r|rrrr} -6 & 1 & 6 & 4 & 24 \\ & \downarrow & -6 & 0 & -24 \\ 1 & 0 & 4 & & \text{LoV} \end{array}$$

 $x = \pm 2i$

 $x^2 + 4 = 0$

 $x^2 = -4$

 $x = -6, \pm 2i$

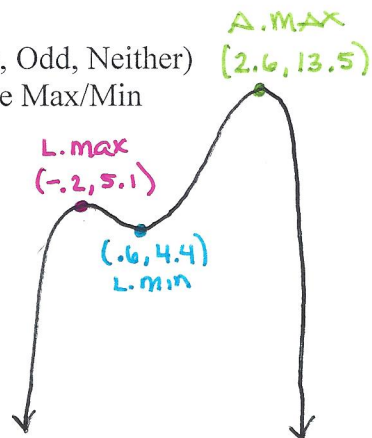
For each of the following equations, answer A-F:

- A) Domain/Range
- B) Name by degree & number of terms
- C) Increasing/Decreasing

- D) End Behavior
- E) Symmetry (Even, Odd, Neither)
- F) Local or Absolute Max/Min

17. $P(x) = -x^4 + 4x^3 - 2x^2 - x + 5$

- A) Domain: \mathbb{R} Range: $y \leq 13.5$
- B) Quartic Polynomial
- C) Increase: $(-\infty, -0.2)$ $(0.6, 2.6)$ Decrease: $(-0.2, 0.6)$ $(2.6, \infty)$
- D) $x \rightarrow -\infty f(x) \rightarrow -\infty$
 $x \rightarrow \infty f(x) \rightarrow -\infty$
- E) Neither
- F) L.max(-0.2, 5.1) L.min(0.6, 4.4) A.max(2.6, 13.5)



18. $P(x) = x^5 - x^4 - 5x^2$

- A) Domain: \mathbb{R} Range: \mathbb{R}
- B) Quintic Trinomial
- C) Increase: $(-\infty, 0)$ $(1.6, \infty)$ Decrease: $(0, 1.6)$
- D) $x \rightarrow -\infty f(x) \rightarrow -\infty$
 $x \rightarrow \infty f(x) \rightarrow \infty$
- E) Neither
- F) L.max(0, 0) L.min(1.6, -8.9)

