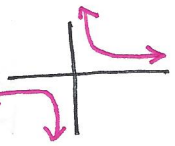
ccelerated Geometry CC
4 Graphing Rational Functions

Name: KEY
Date: _____ **Period:** ___

Rational function – A quotient of 2 Polynomial functions

parent function: $\frac{1}{x}$  2 asymptotes

Domain – x-values being used → All reals OR $x \neq$ ___ (whatever makes denom. 0)

Holes – Factor top & bottom, whatever cancels
Set equal to zero and solve.

Asymptotes – lines that graph approach but NEVER cross

Identifying vertical asymptotes – After you find holes, whatever is left in the denominator set equal to zero

Identifying horizontal asymptotes:

1. If the degree of the numerator is greater that the degree of the denominator then the graph has no horizontal asymptote.
   $\frac{x^3+\ldots}{x^2+\ldots}$ none!
2. If the degree of the numerator is equal to the degree of the denominator and a and b are the leading coefficients of the numerator and the denominator then the horizontal asymptote occurs at $y = \frac{a}{b}$. $\frac{3x^5+\ldots}{2x^5+\ldots}$ $y = 3/2$
3. If the degree of the numerator is less than the degree of the denominator then the x-axis (y = 0) is the horizontal asymptote.
   $\frac{x^2+\ldots}{x^3+\ldots}$ $y=0$

x-int: what's left in the top after finding the holes

y-int: plug in zero for x.

Slant(Oblique) asymptotes -
If the degree of the top is exactly one more than the degree on bottom, then there is a slant. → long division!

*you will never have both a horizontal & a slant!

Give the domain, identify all holes and asymptotes and give the x and y intercepts for each rational function.

1. $f(x) = \frac{x^2 - x - 6}{x^2 + 5x + 6} = \frac{(x-3)(x+2)}{(x+3)(x+2)}$

Domain: $x \neq -3, -2$

Holes: $x = -2$

VA: $x = -3$

HA: $y = 1$

SA: none

x-int: $(3, 0)$

y-int: $(0, -1)$ ← plug 0 in for x.

2. $f(x) = \frac{x+5}{x^2 + 3x - 4} = \frac{(x+5)}{(x+4)(x-1)}$

Domain: $x \neq -4, 1$

Holes: none

VA: $x = -4$ $x = 1$

HA: $y = 0$

SA: none

x-int: $(-5, 0)$

y-int: $(0, -5/4)$ ← plug 0 in for x.

3. $f(x) = \frac{x^2 - 4x + 3}{x^2 - x - 6} = \frac{(x-3)(x-1)}{(x-3)(x+2)}$

Domain: $x \neq 3, -2$

Holes: $x = 3$

VA: $x = -2$

HA: $y = 1$

SA: none

x-int: $(1, 0)$

y-int: $(0, -1/2)$

4. $f(x) = \frac{x^2 + 6x - 7}{x - 1} = \frac{(x+7)(x-1)}{(x-1)}$

Domain: $x \neq 1$

Holes: $x = 1$

VA: none

HA: none

SA: $y = x + 7$

$$x-1 \overline{)\, x^2 + 6x - 7}$$
$$x + 7$$
$$x^2 - x$$
$$7x - 7$$
$$7x - 7$$
$$0$$

x-int: $(-7, 0)$

y-int: $(0, 7)$

5. $f(x) = \frac{5x^2 + 8}{2x^2 - 3x} = \frac{5x^2 + 8}{x(2x-3)}$

Domain: $x \neq 0, 3/2$

Holes: none

VA: $x = 0$ & $x = 3/2$

HA: $y = 5/2$

SA: none

x-int: none → $5x^2 + 8 = 0$

$5x^2 = -8$

$x^2 = -8/5$

(imaginary!)

y-int: none

↑ you get zero in the denominator!

6. $f(x) = \frac{x^2 + 5x - 6}{x + 2} = \frac{(x+6)(x-1)}{(x+2)}$

Domain: $x \neq -2$

Holes: none

VA: $x = -2$

HA: none

SA: $y = x + 3$

$$x+2 \overline{)\, x^2 + 5x - 6}$$
$$x + 3$$
$$x^2 + 2x$$
$$3x - 6$$
$$3x + 6$$
$$-12$$

x-int: $(-6, 0)(1, 0)$

y-int: $(0, -3)$

7. $f(x) = \frac{x+5}{x^2+3x-4} = \frac{(x+5)}{(x+4)(x-1)}$

Domain: $x \neq -4, 1$

Holes: none

VA: $x = -4$ & $x = 1$

HA: $y = 0$

SA: none

x-int: $(-5, 0)$

y-int: $(0, -5/4)$

8. $f(x) = \frac{x+5}{x^2+4x-5} = \frac{(x+5)}{(x+5)(x-1)}$

Domain: $x \neq -5, 1$

Holes: $x = -5$

VA: $x = 1$

HA: $y = 0$

SA: none

x-int: none

y-int: $(0, -1)$

9. $f(x) = \frac{x^2+4x+3}{x^2-9} = \frac{(x+3)(x+1)}{(x-3)(x+3)}$

Domain: $x \neq 3, -3$

Holes: $x = -3$

VA: $x = 3$

HA: $y = 1$

SA: none

x-int: $(0, -1)$

y-int: $(0, -1/3)$

10. $f(x) = \frac{x^2-2x+1}{3x^2-9x+6} = \frac{(x-1)(x-1)}{3(x-2)(x-1)}$

$3(x^2-3x+2)$

Domain: $x \neq 2, 1$

Holes: $x = 1$

VA: $x = 2$

HA: $y = 1/3$

SA: none

x-int: none → $x = 1$ is a Hole so $(1, 0)$ doesn't exist!

y-int: $(0, \frac{1}{6})$

11. $f(x) = \frac{x^2-4}{2x^2+2x-12} = \frac{(x+2)(x-2)}{2(x+3)(x-2)}$

$2(x^2+x-6)$

Domain: $x \neq -3, 2$

Holes: $x = 2$

VA: $x = -3$

HA: $y = 1/2$

SA: none

x-int: $(-2, 0)$

y-int: $(0, 1/3)$

12. $f(x) = \frac{2x^2-3x+1}{x-2} \rightarrow \frac{2x^2-2x-x+1}{x-2}$

$= \frac{2x(x-1)-1(x-1)}{x-2} = \frac{(x-1)(2x-1)}{x-2}$

Domain: $x \neq 2$

Holes: none

VA: $x = 2$

HA: none

SA: $y = 2x+1$

$$\begin{array}{r} 2x+1 \\ x-2 \overline{)\,2x^2-3x+1} \\ 2x^2-4x \\ \hline x+1 \\ x-2 \\ \hline 3 \end{array}$$

x-int: $(1/2, 0)$ & $(1, 0)$

y-int: $(0, -1/2)$