

Polynomials

State the possible rational zeros for each function.

1) $f(x) = x^4 + 6x^2 - 16$
"q" "p"

$\frac{\pm 1 \pm 2 \pm 4 \pm 8 \pm 16}{\pm 1} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

2) $f(x) = 6x^5 + 3x^4 + 16x^3 + 8x^2 + 10x + 5$
"q" "p"

$\frac{\pm 1 \pm 5}{\pm 1 \pm 2 \pm 3 \pm 6} = \pm 1, \pm 5, \pm 1/2, \pm 5/2, \pm 1/3, \pm 5/3, \pm 1/6, \pm 5/6$

Find all zeros.

3) $f(x) = 3x^3 - 5x - 14$ From graph: 2

$$\begin{array}{r|rrrr} 2 & 3 & 0 & -5 & -14 \\ & \downarrow & 6 & 12 & 14 \\ \hline & 3 & 6 & 7 & \text{LoV} \end{array}$$

\downarrow
 $3x^2 + 6x + 7$

$x = \frac{-6 \pm \sqrt{36 - 4(3)(7)}}{2(3)}$

$x = \frac{-6 \pm \sqrt{-48}}{6}$
 $x = \frac{-6 \pm 4i\sqrt{3}}{6}$
 $= -1 \pm \frac{2i\sqrt{3}}{3}$

$x = 2, -1 \pm \frac{2i\sqrt{3}}{3}$

4) $f(x) = 5x^3 + x^2 - 5x - 1$ From graph: -1, 1

$$\begin{array}{r|rrrr} -1 & 5 & 1 & -5 & -1 \\ & \downarrow & -5 & 4 & 1 \\ \hline 1 & 5 & -4 & -1 & \text{LoV} \\ & \downarrow & 5 & 1 & \\ \hline & 5 & 1 & \text{LoV} & \end{array}$$

\downarrow
 $5x + 1 = 0$
 $5x = -1$
 $x = -1/5$

$x = -1, 1, -1/5$

5) $f(x) = 3x^4 - x^3 - 31x^2 - 3x$

From graph: -3, 0

$$\begin{array}{r|rrrr} -3 & 3 & -1 & -31 & -3 \\ & \downarrow & -9 & 30 & 3 \\ \hline 0 & 3 & -10 & -1 & \text{LoV} \\ & \downarrow & 0 & 0 & 0 \\ \hline & 3 & -10 & -1 & \text{LoV} \end{array}$$

\downarrow
 $3x^2 - 10x - 1$

$x = \frac{10 \pm \sqrt{100 - 4(3)(-1)}}{2(3)}$
 $x = \frac{10 \pm \sqrt{112}}{6}$
 $x = \frac{10 \pm 4\sqrt{7}}{6} = \frac{5 \pm 2\sqrt{7}}{3}$

$x = -3, 0, \frac{5 \pm 2\sqrt{7}}{3}$

A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.

6) -5, $-2 + \sqrt{2}$, $-2 - \sqrt{2}$

7) 1, $2 + 3i$, $-2 - 2i$, $2 - 3i$, $-2 + 2i$

8) -2, 4, $1 + 3i$, $1 - 3i$

9) -3, $2i$, $-2i$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

10) -2, 3, -5

$$f(x) = (x+2)(x-3)(x+5)$$

$$f(x) = (x^2 - x - 6)(x+5)$$

$$f(x) = x^3 - x^2 - 6x + 5x^2 - 5x - 30$$

$$f(x) = x^3 + 4x^2 - 11x - 30$$

12) -3, $\sqrt{2}$, $-\sqrt{2}$

$$f(x) = (x+3)(x-\sqrt{2})(x+\sqrt{2})$$

$$f(x) = (x+3)(x^2 - 2)$$

$$f(x) = x^3 + 3x^2 - 2x - 6$$

14) 1, $-3+2i$, $-3-2i$

$$f(x) = (x-1)(x-(-3+2i))(x-(-3-2i))$$

$$f(x) = (x-1)(x+3-2i)(x+3+2i)$$

$$f(x) = (x-1)((x+3)^2 - 4i^2)$$

$$f(x) = (x-1)(x^2 + 6x + 9 + 4)$$

$$f(x) = (x-1)(x^2 + 6x + 13)$$

$$f(x) = x^3 + 6x^2 + 13x - x^2 - 6x - 13$$

$$f(x) = x^3 + 5x^2 + 7x - 13$$

11) 5, $-2+3i$, $-2-3i$

$$f(x) = (x-5)(x-(-2+3i))(x-(-2-3i))$$

$$f(x) = (x-5)(x+2-3i)(x+2+3i)$$

$$f(x) = (x-5)((x+2)^2 - 9i^2)$$

$$f(x) = (x-5)(x^2 + 4x + 4 + 9)$$

$$f(x) = (x-5)(x^2 + 4x + 13)$$

$$f(x) = x^3 + 4x^2 + 13x - 5x^2 - 20x - 65$$

$$f(x) = x^3 - x^2 - 7x - 65$$

13) 2, $2+\sqrt{7}$, $2-\sqrt{7}$

$$f(x) = (x-2)(x-(2+\sqrt{7}))(x-(2-\sqrt{7}))$$

$$f(x) = (x-2)(x-2-\sqrt{7})(x-2+\sqrt{7})$$

$$f(x) = (x-2)((x-2)^2 - 7)$$

$$f(x) = (x-2)(x^2 - 4x + 4 - 7)$$

$$f(x) = (x-2)(x^2 - 4x - 3)$$

$$f(x) = x^3 - 4x^2 - 3x - 2x^2 + 8x + 6$$

$$f(x) = x^3 - 6x^2 + 5x + 6$$

15) $1+3i$, $3i$, $1-3i$, $-3i$

$$f(x) = (x-3i)(x+3i)(x-(1+3i))(x-(1-3i))$$

$$f(x) = (x^2 - 9i^2)(x-1-3i)(x-1+3i)$$

$$f(x) = (x^2 + 9)((x-1)^2 - 9i^2)$$

$$f(x) = (x^2 + 9)(x^2 - 2x + 1 + 9)$$

$$f(x) = (x^2 + 9)(x^2 - 2x + 10)$$

$$f(x) = x^4 - 2x^3 + 10x^2 + 9x^2 - 18x + 90$$

$$f(x) = x^4 - 2x^3 + 19x^2 - 18x + 90$$

#11, 13, 14, and 15 done the "easy way"

11. $5, -2+3i \rightarrow x = -2 \pm 3i$

\downarrow
 $(x-5)$ $x+2 = \pm 3i$
 $(x+2)^2 = 9i^2$

\swarrow $x^2+4x+4 = -9$

$(x-5)(x^2+4x+13)$

$f(x) = x^3 + 4x^2 + 13x - 5x^2 - 20x - 65$

$f(x) = x^3 - x^2 - 7x - 65$

13. $2, 2 + \sqrt{7} \rightarrow x = 2 \pm \sqrt{7}$

\downarrow
 $(x-2)$ $x-2 = \pm \sqrt{7}$
 $(x-2)^2 = 7$

\swarrow $x^2-4x+4 = 7$

$(x-2)(x^2-4x-3)$

$f(x) = x^3 - 4x^2 - 3x - 2x^2 + 8x + 6$

$f(x) = x^3 - 6x^2 + 5x + 6$

14. $1, -3+2i \rightarrow x = -3 \pm 2i$

\downarrow
 $(x-1)$ $x+3 = \pm 2i$
 $(x+3)^2 = 4i^2$

\swarrow $x^2+6x+9 = -4$

$(x-1)(x^2+6x+13)$

$f(x) = x^3 + 6x^2 + 13x - x^2 - 6x - 13$

$f(x) = x^3 + 5x^2 + 7x - 13$

15. $3i, 1+3i \rightarrow x = 1 \pm 3i$

\downarrow
 $(x-3i)(x+3i)$ $x-1 = \pm 3i$
 $(x-1)^2 = 9i^2$

$x^2+3ix-3ix-9i^2$ $x^2-2x+1 = -9$

$(x^2+9)(x^2-2x+10)$

$f(x) = x^4 - 2x^3 + 10x^2 + 9x^2 - 18x + 90$

$f(x) = x^4 - 2x^3 + 19x^2 - 18x + 90$

Find all zeros. One zero has been given.

16) $f(x) = 3x^4 + 4x^3 - 39x^2 - 76x - 12; -2$

From graph: -3

$$\begin{array}{r|rrrrr} -2 & 3 & 4 & -39 & -76 & -12 \\ & \downarrow & -6 & 4 & 70 & 12 \\ \hline -3 & 3 & -2 & -35 & -6 & \text{LoV} \\ & \downarrow & -9 & 33 & 6 & \\ \hline & 3 & -11 & -2 & \text{LoV} & \end{array}$$

$$3x^2 - 11x - 2$$

$$x = \frac{11 \pm \sqrt{121 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{11 \pm \sqrt{145}}{6}$$

$$x = -2, -3, \frac{11 \pm \sqrt{145}}{6}$$

From graph: -3, 1

17) $f(x) = 9x^4 + 30x^3 - 8x^2 - 46x + 15; -\frac{5}{3}$

$$\begin{array}{r|rrrrr} -5/3 & 9 & 30 & -8 & -46 & 15 \\ & \downarrow & -15 & -25 & +55 & -15 \\ \hline -3 & 9 & 15 & -33 & 9 & \text{LoV} \\ & \downarrow & -27 & 36 & -9 & \\ \hline & 9 & -12 & 3 & \text{LoV} & \\ & \downarrow & 9 & -3 & & \\ \hline & 9 & -3 & \text{LoV} & & \end{array}$$

$$x = -3, -1, 1/3, -5/3$$

$$9x - 3 = 0$$

$$9x = 3$$

$$x = 1/3$$

18) $f(x) = 4x^5 - 10x^4 - 14x^3 + 35x^2 + 10x - 25; \frac{5}{2}$

From graph: -1, 1

$$\begin{array}{r|rrrrrr} 5/2 & 4 & -10 & -14 & 35 & 10 & -25 \\ & \downarrow & 10 & 0 & -35 & 0 & 25 \\ \hline -1 & 4 & 0 & -14 & 0 & 10 & \text{LoV} \\ & \downarrow & -4 & 4 & 10 & -10 & \\ \hline & 4 & -4 & -10 & 10 & \text{LoV} & \\ & \downarrow & 4 & 0 & -10 & & \\ \hline & 4 & 0 & -10 & \text{LoV} & & \end{array}$$

$$4x^2 + 0x - 10$$

$$x = \frac{-0 \pm \sqrt{0 - 4(4)(-10)}}{2(4)}$$

$$x = \frac{\pm \sqrt{160}}{8} = \frac{\pm 4\sqrt{10}}{8} = \frac{\pm \sqrt{10}}{2}$$

$$x = 5/2, -1, 1, \frac{\pm \sqrt{10}}{2}$$

Factor each and find all roots.

19) $x^3 - x^2 - 12x = 0$

$$x(x^2 - x - 12) = 0$$

$$x(x-4)(x+3) = 0$$

$$x = 0, 4, -3$$

20) $x^4 - 4x^2 - 21 = 0$

$$(x^2 - 7)(x^2 + 3) = 0$$

$$\begin{array}{l} \downarrow \\ x^2 - 7 = 0 \\ x^2 = 7 \\ x = \pm\sqrt{7} \end{array} \quad \begin{array}{l} \downarrow \\ x^2 + 3 = 0 \\ x^2 = -3 \\ x = \pm i\sqrt{3} \end{array}$$

$$x = \pm\sqrt{7}, \pm i\sqrt{3}$$

$$21) x^4 - 11x^2 + 28 = 0$$

$$(x^2 - 7)(x^2 - 4) = 0$$

$$\downarrow$$

$$x^2 - 7 = 0$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

$$\downarrow$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

or

$$(x^2 - 7)(x - 2)(x + 2)$$

$$x = \pm\sqrt{7}, \pm 2$$

$$23) x^3 - 4x^2 - 5x = 0$$

$$x(x^2 - 4x - 5) = 0$$

$$x(x - 5)(x + 1) = 0$$

$$x = 0, 5, -1$$

$$22) x^4 - 7x^2 + 12 = 0$$

$$(x^2 - 4)(x^2 - 3) = 0$$

$$\downarrow$$

$$(x - 2)(x + 2)(x^2 - 3) = 0$$

$$\downarrow$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$$x = 2, -2, \pm\sqrt{3}$$

$$24) x^3 - 5x^2 + 4x = 0$$

$$x(x^2 - 5x + 4) = 0$$

$$x(x - 4)(x - 1) = 0$$

$$x = 0, 4, 1$$

$$25) (x^3 + 5x^2 - 3x - 15) = 0$$

$$x^2(x + 5) - 3(x + 5) = 0$$

$$(x + 5)(x^2 - 3) = 0$$

$$\downarrow$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$$x = -5, \pm\sqrt{3}$$

$$26) (x^6 + 2x^4 - 16x^2 - 32) = 0$$

$$x^4(x^2 + 2) - 16(x^2 + 2) = 0$$

$$(x^2 + 2)(x^4 - 16) = 0$$

$$(x^2 + 2)(x^2 - 4)(x^2 + 4) = 0$$

$$(x^2 + 2)(x - 2)(x + 2)(x^2 + 4) = 0$$

$$\downarrow$$

$$x^2 + 2 = 0$$

$$x^2 = -2$$

$$x = \pm i\sqrt{2}$$

$$\downarrow$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

$$x = \pm i\sqrt{2}, 2, -2, \pm 2i$$