

Solve each polynomial equation by factoring.

$$1. 5x^3 + 10x^2 + 5x = 0$$

$$5x(x^2 + 2x + 1) = 0$$

$$5x(x+1)(x+1) = 0 \quad x = 0, x = -1$$

$$3. 4x^3 + 40x^2 + 100x = 0$$

$$4x(x^2 + 10x + 25) = 0$$

$$4x(x+5)(x+5) = 0 \quad x = 0, -5$$

$$2. (x^3 + 2x^2)(-9x - 18) = 0$$

$$x^2(x+2)(-9(x+2)) = 0$$

$$(x+2)(x^2-9) = 0 \rightarrow (x+2)(x+3)(x-3) = 0$$

$$x = -2, -3, 3$$

$$4. 8x^4 - 48x^3 = 56x^2$$

$$8x^4 - 48x^3 - 56x^2 = 0$$

$$8x^2(x^2 - 6x - 7) = 0 \rightarrow 8x^2(x-7)(x+1) = 0$$

$$x = 0, 7, -1$$

Identify the roots of each equation. State the multiplicity of each root.

$$5. x^3 + 2x^2 - 3x = 0 \quad x = -3 \quad x = 0 \quad x = 1$$

multiplicity of 1

$$1(x+3)(x-0)(x-1)$$

$$= 1x(x+3)(x-1)$$

$$7. x^3 - 7x^2 + 11x - 5 = 0 \quad x = 1 \quad x = 5$$

mult. of 2 mult. of 1

$$1(x-1)^2(x-5)$$

$$= 1(x-1)^2(x-5)$$

$$6. 2x^3 - 4x^2 + 2x = 0 \quad x = 0 \quad x = 1$$

mult. of 1 mult. of 2

$$2(x-0)(x-1)^2$$

$$= 2x(x-1)^2$$

$$8. x^3 + 12x^2 + 36x = 0 \quad x = 0 \quad x = -6$$

mult. of 1 mult. of 2

$$1(x-0)(x+6)^2$$

$$= 1x(x+6)^2$$

Use the Rational Root Theorem to identify all possible rational roots.

$$9. 3x^3 + 2x^2 - 1 = 0 \quad \frac{\pm 1}{\pm 1 \pm 3} = \pm 1, \pm 1/3$$

$$10. x^4 - 3x^2 + 12 = 0 \quad \frac{\pm 1 \pm 2 \pm 3 \pm 4 \pm 6 \pm 12}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$\pm 1, \pm 1/3$$

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$11. 2x^3 - x - 10 = 0 \quad \frac{\pm 1 \pm 2 \pm 5 \pm 10}{\pm 1 \pm 2} = \pm 1, \pm 2, \pm 5, \pm 10, \pm 5/2, \pm 1/2$$

$$12. x^4 + 6x - 9 = 0 \quad \frac{\pm 1 \pm 3 \pm 9}{\pm 1} = \pm 1, \pm 3, \pm 9$$

$$\pm 1, \pm 2, \pm 5, \pm 10, \pm 5/2, \pm 1/2$$

$$\pm 1, \pm 3, \pm 9$$

$$2x^3 - 9x^2 + 2 = 0 \quad \frac{\pm 1 \pm 2}{\pm 1 \pm 2} = \pm 1, \pm 2, \pm 1/2$$

From graph: $1/2 \leftarrow$ check with synthetic

$$\begin{array}{r|rrrr} 1/2 & 2 & -9 & 0 & 2 \\ & \downarrow & 1 & -4 & -2 \\ \hline & 2 & -8 & -4 & 0 \end{array}$$

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$$2x^2 - 8x - 4 = 0$$

$$x = \frac{-(-8) \pm \sqrt{64 - 4(2)(-4)}}{2(2)} = \frac{8 \pm \sqrt{96}}{4} = \frac{8 \pm 4\sqrt{6}}{4} = 2 \pm \sqrt{6}$$

$$x = 1/2, 2 \pm \sqrt{6}$$